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Crossing heavy-flavour thresholds in fragmentation functions

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Abstract: In analogy with parton distribution functions, also parton fragmentation functions obey matching conditions when crossing heavy-flavour thresholds. We compute these matching conditions at next-to-leading order in the strong coupling constant α_S in the $\overline{\mathrm{MS}}$ scheme. Our results can be used for the dynamical generation of the heavy-flavour component in next-to-leading order fits to light-hadrons fragmentation functions. Furthermore, when computing perturbatively the charm fragmentation function from first principles and evolving it to higher scales, our matching conditions should be used for consistency when crossing the bottom threshold.

Keywords: NLO Computations, QCD.

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Contents

1. Introduction

In full analogy with structure functions, also fragmentation functions change when heavy-flavour thresholds are crossed. In a system with $n_{\rm L}$ light flavours, and a heavy flavour of mass m, the inclusive production of a hadron at a scale much below the heavy-flavour threshold is given by

$$\frac{d\sigma}{dp_{\rm T}} = \int dz\, d\hat{p}_{\rm T} \sum_{i\in\mathbb{I}_{n_{\rm L}}} D_i^{(n_{\rm L})}(z,\mu) \frac{d\hat{\sigma}_i^{(n_{\rm L})}(\mu)}{d\hat{p}_{\rm T}} \delta(p_{\rm T} - z\hat{p}_{\rm T})\,, \tag{1.1}$$

where $\mathbb{I}_{n_{\rm L}}$ is the set of all light partons

$$\mathbb{I}_{n_{\rm L}} = \{q_1, \bar{q}_1, \dots q_{n_{\rm L}}, \bar{q}_{n_{\rm L}}, g\}\,, \tag{1.2}$$

$d\hat{\sigma}_i^{(n_{\rm L})}/d\hat{p}_{\rm T}$ are the short distance cross sections, calculable in perturbation theory, and $D_i^{(n_{\rm L})}$ are the partonic fragmentation functions that describe the hadronization of the light partons i into the hadron. The partonic fragmentation functions $D_i^{(n_{\rm L})}$ are normally not calculable perturbatively, but their evolution in terms of the scale μ is given by the Altarelli-Parisi equations with $n_{\rm L}$ flavours.

Renormalization is performed in the so called decoupling scheme [1]: divergences associated with light particles are subtracted in the $\overline{\rm MS}$ scheme, while divergent heavy-quark loop effects are subtracted at zero momentum. This results in the fact that, for momenta much below the heavy-quark mass, the heavy-quark existence can be completely ignored. In the following, we will also refer to this scheme as the $n_{\rm L}$ scheme.

Much above the heavy-flavour threshold, an equation of the form (1.1) also holds, but with $n_{\rm L}$ replaced by $n = n_{\rm L} + 1$. The $D_i^{(n)}$ obey Altarelli-Parisi equations for n flavours, and the standard $\overline{\rm MS}$ scheme is used for all flavours. We will refer to this scheme as the n scheme.

For a heavy enough quark, the relation of $D^{(n)}$ with $D^{(n_{\rm L})}$ can be computed in perturbative QCD. At the leading logarithmic level, we have

$$D_i^{(n)}(z,\mu) = \begin{cases} D_i^{(n_{\rm L})}(z,\mu) + \mathcal{O}(\alpha_{\rm S}) & \text{for } i \in \mathbb{I}_{n_{\rm L}} \\ \mathcal{O}(\alpha_{\rm S}) & \text{for } i \in \{h, \bar{h}\} \end{cases} \qquad \text{when } \mu \approx m\,, \tag{1.3}$$

JHEP10(2005)034

where we have introduced the notation $h = q_n$ for the heavy flavour. The matching condition (1.3) simply states that the presence of the heavy quark has effects of order $\alpha_{\rm S}$ in processes at scales near its mass. A relation of the same form as (1.3) holds for parton densities below and above a heavy-flavour threshold. In this case, however, the next-to-leading order (NLO) accurate matching condition has also been known for a long time [2]. It requires $D_h^{(n)}$ to be of order $\alpha_{\rm S}^2$ and the difference of $D^{(n)}$ and $D^{(n_{\rm L})}$ for the light flavours to be at most of order $\alpha_{\rm S}^2$, for μ exactly equal to m. This matching condition is commonly used in next-to-leading logarithmic (NLL) parton distribution functions fits, where the heavy-quark distributions $c(x,\mu)$ and $b(x,\mu)$ are radiatively generated, rather than directly fitted to the data.

For fragmentation functions, NLO matching has never appeared in the literature, and it has never been used in global fits. Usually [3−8] the heavy-flavour fragmentation functions are simply parametrized and fitted to the data. In [9], they are instead generated dynamically, but using only the leading-order matching condition (1.3).

In the present work, we compute the matching conditions for fragmentation functions at the NLO level in the $\overline{\rm MS}$ scheme. The NLO matching conditions should be used for consistency in NLL fragmentation-function fits that generate heavy flavours dynamically.[1] In particular, in the computation of the charm fragmentation function from first principles [10], the matching conditions should be used when crossing the bottom threshold. In this framework, at low energies (i.e. not much above the charm mass), the charm is treated as a heavy quark, in order to provide a perturbative expression for its fragmentation function. Near the bottom threshold, the bottom is treated as heavy, while all other quarks (including charm) are considered light.

2. Matching conditions at NLO accuracy

We consider a light-hadron H inclusive production process in e^+e^- annihilation[2]

$$e^+e^- \rightarrow H + X \tag{2.1}$$

at a center-of-mass energy Q much above the heavy-flavour mass, so that power-suppressed m/Q terms can be neglected, but not too large, so that $\alpha_S \log(m/Q) \ll 1$.

In this regime, we can perform the calculation in both the decoupling and the full $\overline{\rm MS}$ scheme. In the first scheme, we can neglect to resum large logarithms of m/Q, since $\alpha_S \log(m/Q)$ is small, use the $D^{(n_{\rm L})}$ fragmentation functions, and include the heavy-flavour effects by fixed $\mathcal{O}(\alpha_{\rm S})$ calculation. On the other hand, in the second scheme, we can neglect power-suppressed m/Q terms, treat all flavours as light, and use the full $\overline{\rm MS}$ scheme, with the $D^{(n)}$ fragmentation functions and the corresponding factorization formulae.

[1] One should note that light hadrons can be produced by heavy quarks not only via fragmentation effects, like the ones considered in this work, but also directly via weak decays. Such contributions can be properly modeled, and should be taken into account when fitting the experimental data, but they are physically disjointed from the strong interaction problem addressed here, as they happen on much longer time scales.

[2] Universality of the fragmentation function guarantees that the result obtained in a specific process remains valid for all processes.

JHEP10(2005)034

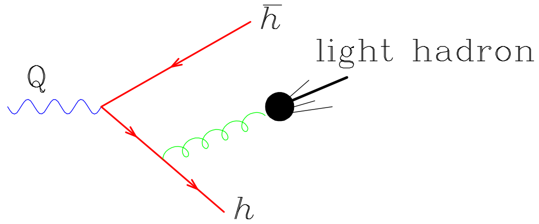

Figure 1: Heavy-flavour contribution to the production of a light hadron.

In the decoupling scheme, we can write the differential cross section for the production of a light hadron H with energy fraction $x = 2E_H/Q$ as

$$\frac{d\sigma}{dx} = \int_x^1 \frac{dy}{y} \left\{ \sum_{i \in \mathbb{I}_{n_\mathrm{L}}} D_i^{(n_\mathrm{L})}\left(\frac{x}{y}, \mu\right) \frac{d\hat{\sigma}_i^{(n_\mathrm{L})}(y,\mu)}{dy} + D_g^{(n_\mathrm{L})}\left(\frac{x}{y}, \mu\right) \frac{d\sigma_{h\bar{h}g}(y)}{dy} \right\}. \tag{2.2}$$

The first term involves the fragmentation function and the short distance cross sections $\hat{\sigma}_i$ for all partons i, excluding the heavy flavour. The second term is the $\mathcal{O}(\alpha_\mathrm{S})$ heavy-flavour contribution, arising from direct production of a heavy-flavoured pair, followed by gluon radiation and fragmentation, as depicted in figure 1. The $\sigma_{h\bar{h}g}$ cross section can be easily obtained from the appendix of ref. [11]

$$\frac{d\sigma_{h\bar{h}g}(y)}{dy} = \sigma_{h\bar{h}} \frac{\alpha_\mathrm{S}}{2\pi} C_\mathrm{F}\, 2\frac{1+(1-y)^2}{y} \left\{ \log\frac{Q^2}{m^2} + \log(1-y) - 1 \right\}, \tag{2.3}$$

where $\sigma_{h\bar{h}}$ is the Born cross section for the production of a heavy quark-antiquark pair, $y = 2E_g/Q$ is the energy fraction carried by the gluon, and $C_\mathrm{F} = 4/3$. Terms suppressed by powers of m/Q have been consistently neglected in eq. (2.3).

The calculation for the same observable $d\sigma/dx$ can also be performed using the full (massless) $\overline{\mathrm{MS}}$ scheme, treating all flavours as massless. Defining $\mathbb{I}_n = \mathbb{I}_{n_\mathrm{L}} \cup \{h, \bar{h}\}$, we obtain

$$\frac{d\sigma}{dx} = \int_x^1 \frac{dy}{y} \sum_{i \in \mathbb{I}_n} D_i^{(n)}\left(\frac{x}{y}, \mu\right) \frac{d\hat{\sigma}_i^{(n)}(y,\mu)}{dy}. \tag{2.4}$$

The difference between the expressions (2.2) and (2.4) yields

$$\begin{aligned}
&\int_x^1 \frac{dy}{y} \sum_{i \in \mathbb{I}_{n_\mathrm{L}},\, i \neq g} \left[D_i^{(n)}\left(\frac{x}{y}, \mu\right) - D_i^{(n_\mathrm{L})}\left(\frac{x}{y}, \mu\right) \right] \frac{d\hat{\sigma}_i(y,\mu)}{dy} + \\
&+ \int_x^1 \frac{dy}{y} \left[D_g^{(n)}\left(\frac{x}{y}, \mu\right) \frac{d\hat{\sigma}_g^{(n)}(y,\mu)}{dy} - D_g^{(n_\mathrm{L})}\left(\frac{x}{y}, \mu\right) \frac{d\hat{\sigma}_g^{(n_\mathrm{L})}(y,\mu)}{dy} \right] + \\
&+ \int_x^1 \frac{dy}{y} \left[\sum_{i \in \{h,\bar{h}\}} D_i^{(n)}\left(\frac{x}{y}, \mu\right) \frac{d\hat{\sigma}_i(y,\mu)}{dy} - D_g^{(n_\mathrm{L})}\left(\frac{x}{y}, \mu\right) \frac{d\sigma_{h\bar{h}g}(y)}{dy} \right] = 0.
\end{aligned} \tag{2.5}$$

In the first term of eq. (2.5) we have replaced $\hat{\sigma}_i^{(n)}$ and $\hat{\sigma}_i^{(n_\mathrm{L})}$ with $\hat{\sigma}_i$, since these cross sections only differ by terms of order α_S^2. In the second term, since $\hat{\sigma}_g$ is of order α_S, we can consider $D_g^{(n_\mathrm{L})} = D_g^{(n)}$, since their difference, according to eq. (1.3), is of order α_S.

Furthermore

$$\frac{d\hat{\sigma}_g^{(n)}(y,\mu)}{dy} - \frac{d\hat{\sigma}_g^{(n_{\rm L})}(y,\mu)}{dy} = \frac{d\hat{\sigma}_{h\bar{h}g}(y,\mu)}{dy}\,, \tag{2.6}$$

where $d\hat{\sigma}_{h\bar{h}g}(y,\mu)/dy$ is the massless $\overline{\rm MS}$-subtracted cross section for the process of figure 1, i.e. a gluon emitted from the (now only nominally) heavy quark.

Finally, in the third term of eq. (2.5), $D^{(n)}_{h/\bar{h}}$ are of order $\alpha_{\rm S}$ (according to eq. (1.3)), so that we only need the Born term for the hard cross section

$$\frac{d\hat{\sigma}_{h/\bar{h}}(y,\mu)}{dy} = \sigma_{h\bar{h}}\,\delta(1-y) + \mathcal{O}(\alpha_{\rm S})\,. \tag{2.7}$$

Since eq. (2.5) must hold for arbitrary electric charges of each quark flavour, it follows immediately that, neglecting terms of order $\alpha_{\rm S}^2$,

$$D_i^{(n)}(x,\mu) = D_i^{(n_{\rm L})}(x,\mu) \qquad \text{for } i \in \mathbb{I}_{n_{\rm L}},\ i \neq g\,. \tag{2.8}$$

We thus find

$$\int_x^1 \frac{dy}{y} D_g\left(\frac{x}{y},\mu\right)\left[\frac{d\hat{\sigma}_{h\bar{h}g}(y,\mu)}{dy} - \frac{d\sigma_{h\bar{h}g}(y)}{dy}\right] + \int_x^1 \frac{dy}{y} \sum_{i\in\{h,\bar{h}\}} D_i^{(n)}\left(\frac{x}{y},\mu\right)\frac{d\hat{\sigma}_i(y,\mu)}{dy} = 0\,, \tag{2.9}$$

which implies

$$D_h^{(n)}(x,\mu) + D_{\bar{h}}^{(n)}(x,\mu) = \frac{1}{\sigma_{h\bar{h}}}\int_x^1 \frac{dy}{y}\left[\frac{d\sigma_{h\bar{h}g}(y)}{dy} - \frac{d\hat{\sigma}_{h\bar{h}g}(y,\mu)}{dy}\right] D_g\left(\frac{x}{y},\mu\right). \tag{2.10}$$

The $\overline{\rm MS}$ subtracted cross section $\hat{\sigma}_{h\bar{h}g}(y,\mu)$ is given by [11, 12]

$$\frac{d\hat{\sigma}_{h\bar{h}g}(y,\mu)}{dy} = \sigma_{h\bar{h}}\,\frac{\alpha_{\rm S}}{2\pi}\,C_{\rm F}2\frac{1+(1-y)^2}{y}\left\{2\log y + \log(1-y) + \log\frac{Q^2}{\mu^2}\right\}, \tag{2.11}$$

and thus, using eqs. (2.3) and (2.11), we obtain

$$D_h^{(n)}(x,\mu) = D_{\bar{h}}^{(n)}(x,\mu) = \int_x^1 \frac{dy}{y} D_g\left(\frac{x}{y},\mu\right)\frac{\alpha_{\rm S}}{2\pi}\,C_{\rm F}\,\frac{1+(1-y)^2}{y}\left[\log\frac{\mu^2}{m^2} - 1 - 2\log y\right]. \tag{2.12}$$

Equations (2.8) and (2.12) are the matching conditions for the D_q and D_h, accurate up to terms of order $\alpha_{\rm S}$. We remark that the right hand side in eq. (2.12) does *not* vanish for $\mu = m$. Under this respect, matching conditions for parton fragmentation functions differ from those for distribution functions [2], which instead vanish at the heavy-quark threshold.

Contrary to D_q and D_h, the matching condition for the gluon fragmentation function D_g has not yet been determined to NLO accuracy. In fact, in e^+e^- annihilation, we are only sensitive to the leading order value of D_g, since gluon production is suppressed by a power of $\alpha_{\rm S}$. On the other hand, if we consider a wide evolution span from m to Q, such that $\alpha_{\rm S}\log(Q/m) \approx 1$, an $\mathcal{O}(\alpha_{\rm S})$ error in D_g at m would propagate into an $\mathcal{O}(\alpha_{\rm S})$ error in

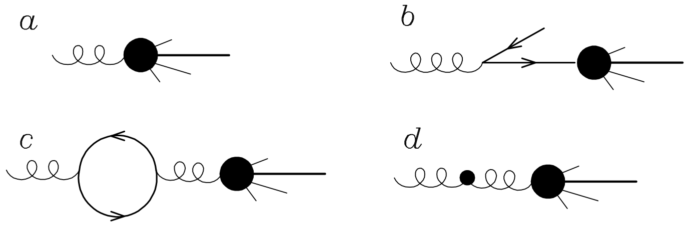

Figure 2: Contributions to the production of a light hadron from a gluon.

all the other components of D through evolution. We must therefore provide a matching condition for D_g accurate to order $\alpha_{\rm S}$. We will show that, in fact,

$$D_g^{(n)}(z,\mu) = D_g^{(n_{\rm L})}(z,\mu) + \mathcal{O}(\alpha_{\rm S}^2) \qquad \text{for } \mu = m\,, \tag{2.13}$$

which is the same result that holds for parton densities [2]. In order to prove eq. (2.13), we consider the process of light-hadron production from a gluon. We can imagine that the gluon is produced in some physical process. For concreteness, we can think of a "super-heavy" quark produced in e^+e^- annihilation (renormalized in the same way in the $n_{\rm L}$ and n scheme), that radiates a gluon. The gluon produces a light hadron by fragmentation, according to the graph a in figure 2. We now want to consider the difference in the radiative corrections to this process in the $n_{\rm L}$ and in the n scheme. It is clear that processes where the gluon splits into a pair of light partons give the same contribution in both schemes. These are in fact processes of order $\alpha_{\rm S}$, diagrammatically identical in the $n_{\rm L}$ and n scheme. In these processes (that carry an extra power of $\alpha_{\rm S}$) the strong coupling constant and the fragmentation functions can be taken at leading order, and therefore they coincide, at $\mathcal{O}(\alpha_{\rm S})$, in the two schemes. The process $g \to h\bar{h}$ followed by the fragmentation of the heavy quark (shown in graph b of the figure) exists only in the n scheme, but it is suppressed by $\alpha_{\rm S}^2$, since the fragmentation function D_h is of order $\alpha_{\rm S}$, and therefore does not give a relevant contribution. The only corrections that can contribute to the difference are shown in graphs c and d. In c, a heavy quark loop corrects the gluon propagator. We include in c also the corresponding renormalization counterterm. Graph d represents the difference in the collinear subtractions in the n and $n_{\rm L}$ schemes.

Calling Π the contribution from the heavy-quark loop in graph c, and $C_{\rm r}$ the renormalization counterterm, we have

$$\Pi^{(n_{\rm L})} + C_{\rm r}^{(n_{\rm L})} = 0\,, \tag{2.14}$$

$$\Pi^{(n)} + C_{\rm r}^{(n)} = C_{\rm r}^{(n)} = \frac{1}{\epsilon}\frac{T_{\rm F}\alpha_{\rm S}}{3\pi} \times \text{Born}\,. \tag{2.15}$$

Equation (2.14) follows from the fact that the heavy-quark loop is subtracted at zero momentum, and the gluon is on shell in c. Equation (2.15) follows from the fact that a massless quark loop at zero momentum is zero in dimensional regularization, so only the standard counterterm for a quark loop in the $\overline{\rm MS}$ scheme survives.

For graph d, the difference in the collinear subtraction counterterm arises only from the P_{gg} term of the Altarelli-Parisi splitting functions, which is the only term that depends explicitly upon the number of light flavours. We have

$$-\frac{1}{\epsilon}\frac{\alpha_{\mathrm{S}}}{2\pi}\left[P_{gg}^{(n)}(z)-P_{gg}^{(n_{\mathrm{L}})}(z)\right]=-\frac{1}{\epsilon}\frac{T_{\mathrm{F}}\alpha_{\mathrm{S}}}{3\pi}\,, \tag{2.16}$$

that cancels exactly the remnant from eq. (2.15). The cancellation is in fact obvious. The fermion contribution of the vacuum polarization from a quark loop vanishes, and thus its ultra-violet (UV) component must cancel exactly its infra-red (IR) component. The renormalization counterterm should be equal to the UV component with the opposite sign, and thus equals the IR component. Finally, the collinear counterterm arises precisely from the IR component of a quark loop, and so it compensates the renormalization counterterm.

The only remaining contribution that could differ in the n and n_{L} schemes is the one represented in graph a. It is proportional to the product of the gluon fragmentation function times the coupling α_{S} arising from the emission of the gluon. Since the total result must be scheme independent, we must therefore have

$$\alpha_{\mathrm{S}}^{(n)}(\mu)D_g^{(n)}(z,\mu)=\alpha_{\mathrm{S}}^{(n_{\mathrm{L}})}(\mu)D_g^{(n_{\mathrm{L}})}(z,\mu)\,. \tag{2.17}$$

From the matching condition for the running coupling α_{S}

$$\alpha_{\mathrm{S}}^{(n)}=\alpha_{\mathrm{S}}^{(n_{\mathrm{L}})}\left(1+\frac{T_{\mathrm{F}}\alpha_{\mathrm{S}}}{3\pi}\log\frac{\mu^2}{m^2}\right), \tag{2.18}$$

we therefore infer

$$D_g^{(n)}(z,\mu)=D_g^{(n_{\mathrm{L}})}(z,\mu)\left(1-\frac{T_{\mathrm{F}}\alpha_{\mathrm{S}}}{3\pi}\log\frac{\mu^2}{m^2}\right). \tag{2.19}$$

Summarizing, for $\mu\approx m$ and up to corrections of order α_{S}^2 we have

$$D_{h/\bar{h}}^{(n)}(x,\mu)=\int_x^1\frac{dy}{y}\,D_g\left(\frac{x}{y},\mu\right)\frac{\alpha_{\mathrm{S}}}{2\pi}\,C_{\mathrm{F}}\,\frac{1+(1-y)^2}{y}\left[\log\frac{\mu^2}{m^2}-1-2\log y\right] \tag{2.20}$$

$$D_g^{(n)}(x,\mu)=D_g^{(n_{\mathrm{L}})}(x,\mu)\left(1-\frac{T_{\mathrm{F}}\alpha_{\mathrm{S}}}{3\pi}\log\frac{\mu^2}{m^2}\right) \tag{2.21}$$

$$D_{i/\bar{i}}^{(n)}(x,\mu)=D_{i/\bar{i}}^{(n_{\mathrm{L}})}(x,\mu)\qquad \text{for } i=q_1,\ldots,q_{n_{\mathrm{L}}}\,. \tag{2.22}$$

For future reference, we provide the Mellin transform of $D_{h/\bar{h}}^{(n)}$. Defining

$$D_{h/\bar{h}}^{(n)}(N)\equiv\int_0^1 dx\; x^{N-1}D_{h/\bar{h}}^{(n)}(x,\mu)\,, \tag{2.23}$$

we have

$$D_{h/\bar{h}}^{(n)}(N)=\frac{\alpha_{\mathrm{S}}}{2\pi}\,C_{\mathrm{F}}\left[\frac{2+N+N^2}{N\,(N^2-1)}\left(\log\frac{\mu^2}{m^2}-1\right)+\frac{4}{(N-1)^2}-\frac{4}{N^2}+\frac{2}{(N+1)^2}\right]D_g(N)\,. \tag{2.24}$$

JHEP10(2005)034

3. Conclusions

In this paper we have calculated, at order α_S and in the $\overline{\mathrm{MS}}$ scheme, the matching conditions for parton fragmentation functions at the heavy-quark thresholds. Such conditions can be used to generate radiatively and to next-to-leading accuracy charm and bottom contributions to light-hadron or photon production via fragmentation. This is achieved by evolving light-quarks and gluon fragmentation functions from a low- to a high-energy scale, and through the heavy-quark thresholds.

In a similar fashion, these matching conditions at next-to-leading accuracy are required for full consistency when evolving the perturbatively-calculated charm fragmentation function through the bottom threshold.

The possibility of generating dynamically the heavy-quark contributions to fragmentation will allow to fit light-hadron fragmentation functions with less free parameters than presently done, in full analogy with modern parton distribution functions analyses. We defer these fits to a future paper.

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JHEP10(2005)034